

Q1 Sol:

(i) –

(a) A Stochastic process N_t is a Poisson Process if $N_t \geq N_s$ for every $t > s$ $N_t - N_s \sim \text{Poisson}(\lambda(t-s))$

It has independent increments.

It has Discrete state space & Continuous time frame.

 $N_0 = 0$.

Well explained

(b) A Stochastic process X_t is Compound Poisson process if, $X_t = \sum_{i=0}^{N_t} Y_i$ Where, Y_i 's are IID & N_t is a Poisson process.

Correct definition

(ii) Process A by Time homogeneous Poisson Process as it will not depend on time. It will be time inhomogeneous, please check out the explanation for the same in the answer sheet

Process B by Time homogeneous Poisson Process as it will not depend on time. same issue as in above, it will be time inhomogeneous

Process C by Time inhomogeneous Poisson Process as it will depend on time till last service. It will be time homogenous

Process D by Time inhomogeneous Compound Poisson Process as it will depend on process C. . It will be time homogenous

[3+0]

[3/9]

Poor explanation provided in the (ii) part, make sure to check the original answer once to check how such answers are presented and the amount of explanation required for them

Q2 Sol:

- (i) For Markov Jump process to be time inhomogeneous transition rates between the states must be time dependent.

correct

- (ii) The principal difficulties in modelling Markov Jump process with time inhomogeneous transition rates are:

The extra parameter of time to incorporate increases the cost of the model & it might be very complex & calculative.

There might be difficult to define a set formula to calculate transition rates which is applicable to all time sets.

correct

- (iii) The probability that child is in Good state without jumping to naughty state till time t is the waiting time in good state which follow a exponential distribution with $\lambda = \mu_{GN}(t)$

So, Probability = $\exp(-\int_0^t \mu_{GN}(s) ds)$

= $\exp(-\int_0^t [0.2 + 0.04 s] ds)$

= $\exp(-0.2t - 0.02t^2)$ correct

- (iv) The Let at time t there is at least 50% probability child being Naughty at some time means

$$1 - \exp(-0.2t - 0.02t^2) \leq 0.5$$

$$\exp(-0.2t - 0.02t^2) > 0.5$$

$$0.2t + 0.02t^2 \leq \ln(2)$$

$$0.02t^2 + 0.2t - \ln(2) \leq 0$$

Roots of t is 2.7238 or -12.72

Since, $0 \leq t \leq 8$

So, at $t = 2.7238$ there is at least 50% chance child being Naughty. correct

- (v) We have given all child is in G state at $t = 0$.

$$P_G(t + dt) = P_G(t) P_{GG}(dt) + P_N(t) P_{NG}(dt)$$

Where dt is very small interval of time such that

$$P_{ij}(dt) = dt * \mu_{ij}(t) + o(t) \text{ if } i \text{ not } = j$$

$$\& P_{ii}(dt) = 1 + dt * \mu_{ii}(t) + o(t)$$

$$\text{So, } P_G(t + dt) = P_G(t) * [1 + dt * \mu_{GG}(t) + o(dt)] + P_N(t) * [dt * \mu_{NG}(t) + o(dt)]$$

$$[P_G(t + dt) - P_G(t)]/dt = P_G(t) * \mu_{GG}(t) + P_N(t) * \mu_{NG}(t) + o(dt)/dt$$

By putting $\lim_{dt \rightarrow 0}$,

$$d/dt P_G(t) = P_G(t) * \mu_{GG}(t) + P_N(t) * \mu_{NG}(t)$$

since, $\lim_{dt \rightarrow 0} o(dt)/dt = 0$

$$d/dt P_G(t) = -P_G(t) * [0.2 + 0.02 t] + (1 - P_G(t)) * [0.4 - 0.04 t]$$

since $P_G(t) + P_N(t) = 1$

$$d/dt P_G(t) + P_G(t) * [0.6 - 0.02 t] = 0.4 - 0.04 t$$

the marked portion will only be 0.6

$$[1+1.5+3+2+1.5]$$

$$[9/10]$$

Q3 Sol:

(i) Transition graph:

States: H = Healthy, I = infected by CG, D = death due to CG, M = Death other cause.

$H \rightarrow M = \mu$

$H \rightarrow I = \sigma$

$I \rightarrow D = \tau$

$I \rightarrow M = \rho - \tau$

(ii) Generator Matrix = A

A	H	I	D	M
H	$-(\mu + \sigma)$	σ	0	μ
I	0	$-\rho$	τ	$(\rho - \tau)$
D	0	0	0	0
M	0	0	0	0

correct

Kolmogorov's forward equation for Probability matrix P is

$d/dt P = PA$

(iii) Let W be the tree month in health state & V be the tree month in infected state
& Number of deaths of health trees = X & Number of deaths of CG infected trees = Y

Incomplete

(iv) MLE of τ

Likelihood function, $L = e^{-(V*\tau)} * \tau^Y$

Log likelihood, $\ln(L) = Y * \ln(\tau) - V*\tau$

Differentiate with respect to τ & equate it 0 gives

$\tau^{(est)} = Y/V$

If we double differentiate it with respect to τ then value will less than 0

So, $\tau^{(est)} = Y/V$ is MLE estimate of τ

correct

(v) $\tau^{(est)} = 30/600 = 0.05$

correct

[2+3+1+3+1]

[10/12]

Good attempt

Q4 Sol:

- (i) Probability the Yolanda will remain covered with company C for 5 years = P_{CC}^5
 $= 0.4^5 = 0.01024$ **wrong probability, it will be $0.4^4 = 0.0256$**
- (ii) Let $X(t)$ be the position matrix of Zachary for t th year. & P be the transition matrix given in question.

$$X(2013) = [1 \ 0 \ 0 \ 0]$$

$$X(2014) = X(2013) * P = [0.5, 0.2, 0.2, 0.1]$$

$$X(2015) = X(2014) * P = [0.35, ___]$$

So, Probability that Zachary will not be covered by Company A on 1 Feb 2015 = $1 - 0.35 = 0.65$.

correct

- (iii) Let X be the long-term stationary distribution of the process.

$$X = [A, B, C, D]$$

As per long-term stationary distribution property

$$X = XP$$

$$A = 0.5 A + 0.2 B + 0.3 C \quad \text{correct}$$

$$A = 0.4 B + 0.6 C$$

$$B = 0.2 A + 0.6 B + 0.2 C + 0.2 D = 0.08 B + 0.12 C + 0.6 B + 0.2 C + 0.2 D$$

$$0.32 B = 0.14 C + 0.2 D$$

$$D = 0.1 A + 0.1 B + 0.1 C + 0.6 D$$

$$D = 0.35 B + 0.4 C$$

$$0.32 B = 0.14 C + 0.07 B + 0.08 C$$

$$B = 0.88 C$$

$$D = 0.708$$

$$A = 0.952$$

$$\text{Since, } A + B + C + D = 1$$

$$C = 50/177$$

So, Long term proportions of homeowners in each company is $[0.2689, 0.2486, 0.2825, 0.2]$.

Wrong values

(iv) Transition Matrix after takeover is P'

P'	Adda	B	C
Adda	0.6	0.2	0.2
B	0.3	0.6	0.1
C	0.4	0.2	0.4

correct

(v) The assumption 's appropriateness:

The transition Rates from company D to company A is zero & from other company to company D is also lower so there might be a possibility that Company D might targeting some different group of customers that others are not & which is very different from Company A.

So, there might be a case where assumptions might get true if company A handle the operations of new company very well that they do not loose the customer's interest of both the companies but if not then their might be higher transitions of customers to its remaining competitors.

[0+2+1.5+2+1.5]

[7/12]

Q5 sol:

(i) By using Kolmogorov forward equations.

$$P_{HH}(x, t + dt) = P_{HH}(x, t) * P_{HH}(x + t, dt) + P_{HS}(x, t) * P_{SH}(x + t, dt)$$

Since, not transition out of death state is possible

Let dt be some small point of time such that,

$$P_{ij}(x + t, dt) = dt * \mu_{ij}(t) + o(dt) \quad i \neq j$$

$$\text{Or } P_{ii}(x + t, dt) = 1 + dt * \mu_{ii}(t) + o(dt) \quad i = i$$

So,

$$P_{HH}(x, t + dt) = P_{HH}(x, t) * [1 + dt * \mu_{HH}(t) + o(dt)] + P_{HS}(x, t) * [dt * \mu_{SH}(t) + o(dt)]$$

$$[P_{HH}(x, t + dt) - P_{HH}(x, t)] / dt = P_{HH}(x, t) * \mu_{HH}(t) + P_{HS}(x, t) * \mu_{SH}(t) + o(dt) / dt$$

By putting $\lim_{dt \rightarrow 0}$, LHS become differentiation of $P_{HH}(x, t)$ with respect to t & $o(dt)/dt = 0$

$$\text{So, } d/dt P_{HH}(x, t) = P_{HH}(x, t) * \mu_{HH}(t) + P_{HS}(x, t) * \mu_{SH}(t)$$

$$d/dt P_{HH}(x, t) = P_{HH}(x, t) * [-\sigma(t) - \mu(t)] + P_{HS}(x, t) * \rho(t, C_t)$$

correct

Hence prove

(ii) Probability life remains in healthy state from 0 to $t = \exp[-\int_0^t (\sigma(s) + \mu(s)) ds]$

Needs derivation

(iii) We can integrate Kolmogorov equation from s to t from state i to j as

First jump Assuming the process remains in state i $s + h$ & take first jump at some time $s + h$ from state i to k then goes from k to j in between time t & $s + h$.

Last jump Assuming the process goes from state i to k from time s to $t - h$ & then jump at time $t - h$ in process j from state k & then remain in state j from time $t - h$ to t .

The paragraph you stated above would look better if splitted them into pointers

(iv) Kolmogorov forward integration equation assumes First jump approach while integrating the equation while Kolmogorov forward integration equation assumes last jump approach while integrating the equation correct

(v) Two errors:

(vi) Unattempted

(vii) [5+1+1.5+1+0]

Q6 Sol:

- (i) This process is called Markov chain stochastic process. **correct**
- (ii) –
- (a) Since, no transitions possible from failed state to other state so it is not irreducible process. **correct**
- (b) Since, 1 step transition from state i to i is possible for all states hence, it is aperiodic. **It actually is not aperiodic as it can remain in each state**
- (iii) Let X_t is the position vector of heating element at t^{th} inspection. & P be the transition matrix given.

$$X_0 = [1 \ 0 \ 0 \ 0]$$

$$X_1 = X_0 * P = [0.5 \ 0.2 \ 0.2 \ 0.1]$$

$$X_2 = X_1 * P = [_, _, 0.26, _]$$

$$\text{So, } P_{EP}(2) = 0.26$$

correct

- (iv) Let P' be the new transition matrix

P'	E	G	P
E	0.6	0.2	0.2
G	0.2	0.5	0.3
P	0.5	0	0.5

correct

- (v) Let $X = [A, B, C]$ be long term probability distribution of the heating element states.

$$X = X * P'$$

$$A = 0.5 B + 5/4 C$$

$$B = 0.2 A + 0.5 B$$

$$B = 2/5 A$$

$$C = A$$

$$\text{By } A + B + C = 1$$

$$A = 5/12$$

$$\text{So, Long term distribution} = [5/12, 2/12, 5/12]$$

Wrong equations for long term prob and hence wrong final answers

- (vi) The expected annual cost incurred due to failures of heating element = $100 * 1050 * [5/12 * 0.1 + 2/12 * 0.2 + 5/12 * 0.5] = 29750$.

Correct process but wrong final answer due to error in (v)

- (vii) The new transition matrix

$$P'' = \begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}$$

Long term distribution $X = [A, B]$ is

$$X = X * P''$$

$$A = 0.8A + 0.2 B$$

$$A = B = 0.5$$

So, there are 50%-50% chance of process in each state. Wring

Now, Company having replacing cost on poor state & replacement + wasted cost on direct failure from either state, i.e.

$$\text{New total cost during year} = 100 * \{1050 * [0.5 * 0.1 + 0.5 * 0.2] + 50 * [0.5 * 0.2 + 0.5 * 0.3]\}$$

$$= 17000$$

Hence, reduction in cost of $29750 - 17000 = 12,750$ per year.

Stationary distribution seems to be your weak point, make sure to go back to the concept and try practicing few questions of it as it is very common question in paper

A

$$[1+1+2+2+0.5+1.5+2]$$

$$[10/19]$$

TOTAL:47.5/75

Decent score, but there is a lot of scope of improvement, hence try working on your weak areas

Expecting much better submission in future